### *What is bootstrapping?*

Mathematically speaking, if we uniformly and independently sample from an unknown distribution we recover the same unknown distribution.

### *What does this mean for us?*

We do not know the true distribution of the S&P 500 but we need to use it to get an idea of the type of returns that can occur in the future. Using bootstrapping we can randomly pull pieces of the past (sample historical returns) and string them together to get a possible outcome for the future. This can be done many times to get many possible future S&P 500 returns.

### *How do we generate these possible future returns?*

We use realized historical S&P 500 returns to generate a broad range of possible future S&P 500 returns. Absent reliable estimators of future possible returns, these "bootstraps" serve as our best alternatives.

### *What if the realized historical returns are poor indicators of the future, and the past never repeats itself exactly?*

Bootstrapping has low sensitivity to the results in any particular historical period and it does not restrict testing to only the realized sequence of historical returns.

*How does the sampling process underlying bootstrapping work?*

The key idea is of sampling with replacement. Suppose our sample contains only five observations of S&P 500 returns, represented by five balls labeled A through E.

In this example, we assume the following returns

- Ball A represents a 0.8% return of the S&P 500.
- Ball B represents a -1% return of the S&P 500.
- Ball C represents a -20% return of the S&P 500.
- Ball D represents a 4.5% return of the S&P 500.
- Ball E represents a -3.1% return of the S&P 500.

We put the five balls into a basket and then draw a ball randomly, note its value, and replace it before making another random draw. Our recorded sequence of S&P 500 returns may look like any one of the examples below:

A, B, E, C, D

- This would imply an S&P 500 return of -19.2%.

C, C, C, C, C

- What if C is a period in Fall 2008? The above sequence is one where this worst of 2008 period occurs repeatedly.
- This would imply an S&P 500 return of -67.2%.

C, C, E, E, E

- What if E is a period in Summer 2011 (when the United States lost its AAA rating)? The above sequence is one where this worst of 2008 period occurs twice followed thrice by 2011's market events.
- This would imply an S&P 500 return of -41.8%.

D, D, D, A, D

- This would imply an S&P 500 return of 20.2%.

This method of sampling allows us to test for a wide variety of possible S&P 500 scenarios.